1. Project Number:

2. Title: A Characterization of the Discontinuous Nature of Kriging Digital Terrain Models

3. Focus categories: HYDROL, MOD, G&G

4. Keywords: kriging, digital terrain modeling, surface modeling, surface interpolation, slope, gradient, runoff, continuity, hydrology

5. Duration: March/2001 to February/2002.

6. Federal Funds:

7. Non-federal Funds:

8. Principal Investigator: Thomas H. Meyer, University of Connecticut

9. Congressional District: Second Congressional District.

10. Statement of Critical Regional or State Water Problem:

    Hydrology and surface runoff models depend extensively on digital terrain models (DTM) from which terrain gradients are derived. Surface gradients determine the potential energy available to drive surface flow and is considered by some researchers (Evans 72) to be the most important of all geomorphometrics. Given its critical role in hydrological modeling, it is imperative that hydrologists understand the continuity characteristics of the DTM being employed. The continuity of the DTM is an important contributor to interpolated gradient values, potentially effecting potential energy estimates as well as flow direction. Kriging (Matheron 63, Journal 78, David 77, Goovaerts 97) is a popular choice for the surface interpolation technique used with digital terrain models (DTMs). Bailey (94) asserts that ``there is an argument for kriging to be adopted as a basic method of surface interpolation in GIS as opposed to the standard deterministic tessellation techniques which currently prevail and which can produce artificially smooth surfaces." This argument was discussed in more detail by Laslett (94) who showed that although kriging is mathematically equivalent to minimal energy splines, his study gives an example of a data set for which splines are ``too smooth" and kriging results in more precise estimations. While kriging is not without its critics (Philip and Watson 86a), there is no question that its use is widespread.

    In general, the properties of a mathematical surface being used as a terrain model define the properties imbued to the model. The onus is on the modeler to choose the mathematical surface wisely so that the properties of the surface give the desired traits of the terrain. Continuity properties are of paramount importance. Discontinuous surfaces have ``holes" or ``tears" in them, so to speak, creating fictitious nickpoints (i.e., waterfalls), and zigzagging and broken contour lines. Also, even if a surface were continuous, it might not be altogether smooth, meaning that the surface might have ``creases" in it effecting gradient estimations and flow direction. It is important to catalogue continuity properties and this project will establish these properties for surfaces created with kriging interpolation.

11. Statement of Results or Benefits:

    We will show that DTMs based on kriging are piece-wise discontinuous (zero-order) and then characterize bounds for the discontinuities. We will also determine if there is a correlation with prominent geomorphometrics such as slope, aspect, and roughness. Our goal is to enrich the understanding of hydrologists as to whether kriging provides an acceptable DTM for their purposes. By providing information about the kind and severity of discontinuities one can expect, hydrologists are better equipped to choose among the many types of DTMs currently available for hydrological modeling.

12. Nature, Scope and Objectives of the Research: This is basic research, exploring the mathematical properties of kriging, a widely-used surface interpolator. We focus our attention on the continuity properties of kriging digital terrain models exhibited over a wide range of terrain types. Our goal is to enrich the understanding of hydrologists as to whether kriging provides an acceptable DTM for their purposes. By providing information about the kind and severity of discontinuities one can expect, hydrologists are better equipped to choose among the many types of DTMs currently available for hydrological modeling

13. Methods, Procedures and Facilities: Provide enough information to permit evaluation of the technical adequacy of the approach to satisfy the objectives.

    We will write computer programs to characterize the kriging behaviors over existing USGS 7.5' digital terrain models. These programs will

    - read the input DEM. This is a general study so the DEM should exhibit a wide variety of terrain types, from cliffs to planes. We will analyze the USGS 7.5' DEM of the Sandia Crest quadrangle map, a region in north-central New Mexico. Within this single DEM are sheer cliffs over 500' high, extremely eroded fluvial fans, bajadas, flat tilted uplift zones steeply incised with canyons, low rolling hills, and flat fluvial planes. It is a remarkably complex and diverse landscape in a relatively small area.
    - subdivide it into regions. We envision two subdivision strategies. First, we will subdivide by creating a Voronoi Diagram (Voronoi 1908, Okabe et al 1992) from randomly generated points covering the DEM. The Voronoi polygons will account for all points in the DEM and provide approximately hexagonal regions[1]. Being somewhat circular, this is a good shape from which to compute variograms. Second, if any of the geomorphometrics are strongly correlated to large discontinuities, we plan to subdivide into regions of similar values for that metric instead of subdividing randomly.
    - compute a best variogram model for each subdivision region. Kriging is "best" regarding a variogram model of the variance structure of the terrain samples. The weight matrix is constructed in such a way as to guarantee that it is positive definite, thus guaranteeing it is invertable. In order to guarantee that the weight matrix is positive definite, various models of the variogram are fit to the experimentally computed variogram, see Fig.1 and Fig. 2. These models are chosen so that, when used as a surrogate for the actual data, they will always produce a positive definite weight matrix. There are several variogram models typically used (spherical, exponential, Gaussian, and linear). We will perform a least-squares fit to all four models and automatically select the best one based on r^2 value.

[1] The expected number of sides of a Voronoi polygon generated from randomly distributed points is six.

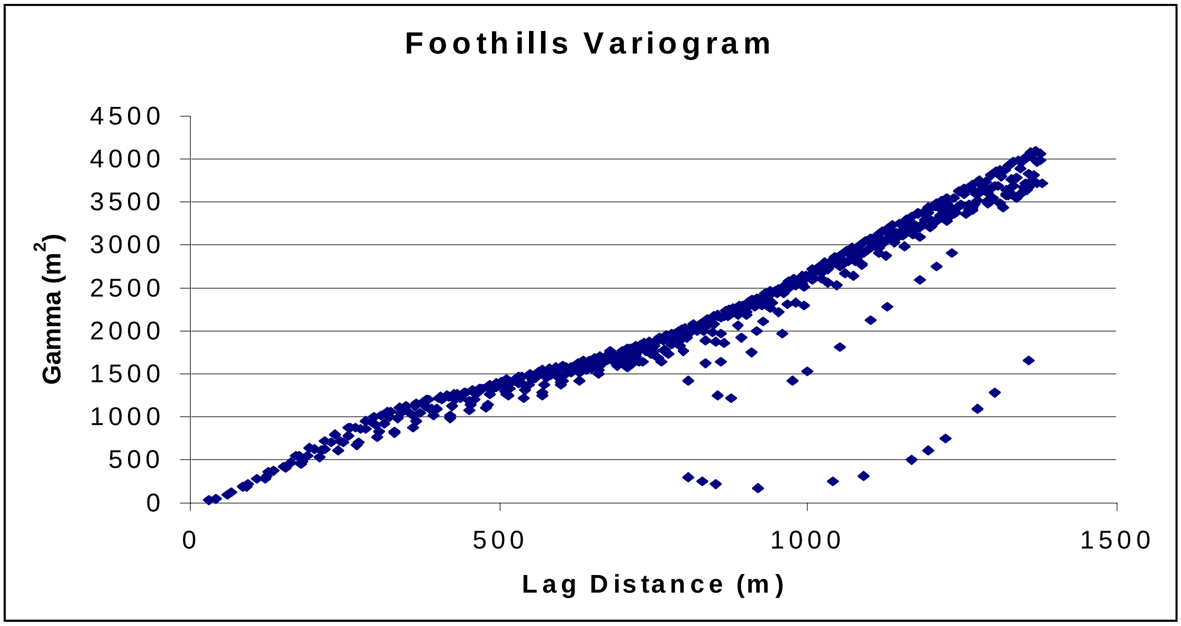

Figure 1.

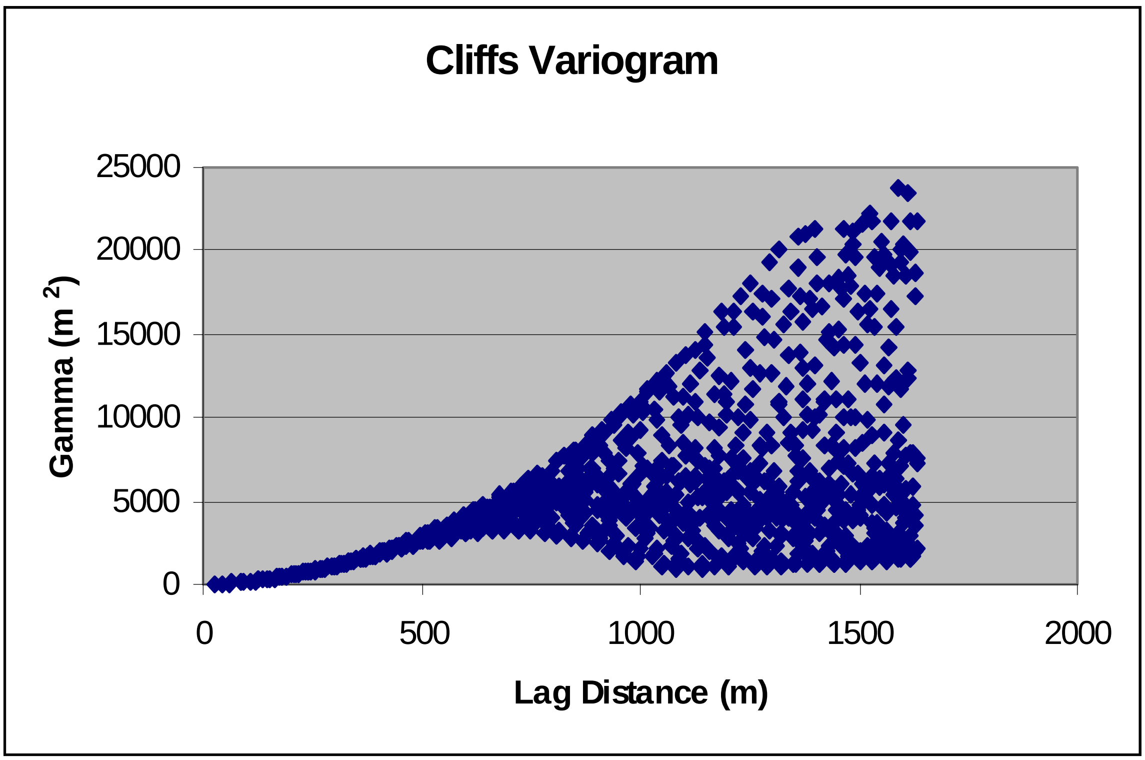

Figure 2.

- use the best variogram model for the kriging interpolation between the DEM elevation postings. We will interpolate all the boundaries between all the polygons within a region to determine the average and maximum discontinuity, along with the variance. The results for each polygon will be tabulated together as a regional description and used for subsequent correlation analyses as described below. We will perform this analysis both for gridded data and for irregularly spaced data (created by omitting points from the DEM).
- compute roughness metrics (Philip and Watson 1986b) for each point in each subdivision region. We hypothesize that roughness will be positively correlated to large discontinuities. Intuitively, this is plausible because necessarily there must be greater topographical variation in rougher

places than smoother ones and such variation could lead to different kriging estimates of the same location based on different neighborhoods.

- compute slope (Meyer et al, in press) for each point in each subdivision region. We hypothesize that slope will be positively correlated to large discontinuities. Intuitively, this is plausible sense because areas of greater slope might have greater topographical variation than relatively flat regions. However, planar topographic surface can be tilted, such as an uplift zone. This study area will provide a good test of this hypothesis because it provides horizontal planar zones in the form of alluvial planes along with flat uplift plates.
- compute aspect (Meyer et al, in press) for each point in each subdivision region. We hypothesize that aspect will be positively correlated to large discontinuities. This is plausible due to the variation in mass wasting caused by the freeze-thaw cycle, a cycle strongly dependent upon aspect. The Sandia Mountains exhibit a large number of landslide zones that may provide insight into this issue.

This research requires no fieldwork. It will be conducted solely on personal computing workstations by writing custom C++ programs.

14. Related Research:

Kriging[2] is a best linear unbiased estimator (BLUE). It is best because one can prove that kriging estimates have the smallest residual variance of any estimator. It is linear because the estimate is a linear combination of the sample values weighted according to their distance and direction from the point of interest. Kriging was derived to give spatially-explicit estimates of subterranean precious metal densities from relatively small sample sets (say, less than 100 core samples). Kriging theory stipulates that the entire data set is to be used in the construction of the weight matrix. The weight matrix is derived by finding the inverse of a matrix with one more row and column than the number of samples (the additional column is a LaGrange multiplier). For example, if one has 100 samples, the weight matrix is derived by inverting a 101x101 matrix. This is manageable with high-speed digital computers. However, digital terrain models usually contain large numbers of samples. For example, a typical USGS 7.5' DEM has approximately 170,000 points in it. If one were to use the kriging theory as derived, one would be obliged to invert an impossibly large matrix to derive the weights. Furthermore, the vast majority of the samples would be too far away to exert any significant influence on the interpolated value. Therefore, the common practice is to select a relatively small subset of points from the DTM that surround the point of interest and use just this neighborhood instead of the entire DEM. This practice causes discontinuities of the interpolated surface, a fact that, to the author's knowledge, is undocumented in the literature.

Kriging has been shown to be equivalent to thin-plate splines (Laslett 94) and, consequently, is smooth within a neighborhood. However, the behavior of kriging on the boundary between neighborhoods has been ignored in the literature. It is easy to prove that the region of overlap between two neighborhoods is discontinuous. We will document this fact and then proceed to examine how bad the discontinuity can be and whether it is more likely to be good or bad according to slope, aspect, and roughness of the topography.

Reference List

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[2] see (Isaaks and Srivastava 1989) for an approachable introduction to kriging and spatial statistics.

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15. Investigator's Qualifications:

16. Training Potential. Estimate the number and level of graduate and undergraduate students, by field of study and degree, that are expected to receive training in this project. See the Training Potential table below.

Field of Study	Academic Level				
	Undergrad	Master's	Ph.D.	Post Ph.D.	Total
Geographic Information Science		1			1

17. Information Transfer (IT) Plan: There is a need for the results of the research programs to be distributed in some form to interested persons within the state and/or region. Therefore, we are requiring that applicants address the issue of technology transfer within the application.

   A. Briefly define the subject matter and the problems to be addressed.

   We will show that DTMs based on kriging are piece-wise discontinuous (zero-order) and then characterize bounds for the discontinuities. The surface continuity structure of a digital terrain model is an important factor in that terrain model's performance in hydrological modeling.

   B. Identify the target audience (academics, town officials, government regulators in health and environmental protection, lake associations, wetland commissions, etc).

   Academics, engineers, hydrologists, environmentalists

   C. Indicate the strategies to be employed (e.g. workshops, seminars, newspaper articles, etc.). The Institute will be requesting short summaries, at a later date, in basic English, for release to the newspapers.

   The primary dissemination mechanism will be publication in conference proceedings and in peer-reviewed journals. However, this material can be presented at the Connecticut Institute for Water Resources' seminar series and other similar seminars around New England, as well.