PROJECT OBJECTIVES

1. To determine the effect of friction in side channels (such as, wash-water troughs or water treatment plant launders, roof gutters, street gutters, and dam spillways) in order to improve design methods.

2. To attempt to analyze the flow behavior in detail so that experimental results may be extrapolated to unusual channel shapes.

ACHIEVEMENT OF OBJECTIVES

Although only one channel configuration was used, it provided a very good understanding of the analytical problem and resulted in an explanation of the poor results from previous experiments that had yielded negative frictional resistance. Methods were developed for separating frictional and impact losses.

RESEARCH PROCEDURES USED

A. Experimental

The single channel configuration is shown in Figure 1. A main open channel of rectangular cross-section 18 inches wide and 30 feet long is joined by a side-inlet channel also 18 inches wide. The main channel has a carefully rounded inlet so that the flow is essentially uniform at the point 5 feet from the inlet where it is joined by the side-channel. The flows can be varied over a wide range in both channels. Flow measurements were made at three places. The main flow passed over a suppressed weir ahead of the channel. The lateral flow was measured by a 90° triangular-notch sharp-edged weir. Total flow could be measured at the outlet by a large volume tank. All units were carefully calibrated. Figure 1 shows the relationship between the two channels and also indicates the measuring stations for
which velocity traverses were made. Speeds and directions for a variety of transverse locations and depths were recorded in addition to surface elevations at each measuring point. A NEMFIC miniature current meter was used to measure velocities. This was attached to a protractor scale and several readings at different angles gave a fair (±5°) indication of direction. Accuracies for evaluating the velocity components seem to be satisfactory when compared to the surface elevations which had to be run several times to obtain acceptable results.

B. Analytical Procedures

The preliminary report from which the original proposal was developed is included as an Appendix since reference to this must be made in order to provide understanding of the results and conclusions from this project.

Section B in the Appendix commences with the time-honored assumption that the energy losses are so high in such flows that it is possible to derive only one equation, such as equation 5, based on momentum. The resulting form mixes the momentum and energy concepts. A side flow entering normal to a main channel must lose its momentum in the process, but a substantial portion of its kinetic energy will turn the corner. It is therefore necessary to develop two separate equations to replace equation 5: a momentum equation similar to equation 5 and an energy equation. In addition to these two equations, it is also useful to look at losses which take place within the flow and away from the solid boundaries. A mixing head loss is used to identify such losses.

Momentum Equation: Using methods developed in References 18, 19, and 20, the following equation is obtained:

$$\frac{dD}{dx} = \frac{S_o - S_f - Q q_x (z \beta - \beta_x)}{1 - \beta Q^2 / g A^2 D}$$

(6)

where:

$$\beta = \int_A u^2 dA / A \bar{u}^2,$$

$$\beta_x = \int_0^{D_B} u_B q_x dD / \bar{u} q_x,$$

$$q_x = \frac{\partial A}{\partial t} + \frac{A}{3x} (A \bar{u}) = q_x (x, t)$$
Where: In steady state: \( (\bar{U}D)_x = \frac{q_x}{2B} \)

and \( \bar{U}D = \frac{1}{2B} \int_0^x q_x \, dx + Q_0 \)

\( q = -V_B \) through the boundary

\( 2B \) is the main channel width

\( u_B \) = velocity component in the \( x \) direction at the boundary of the main channel, \( \pm B \), (see Figure 1).

Other terms are defined with equation 1 through 5 in the Appendix.

Energy Equation: In a manner similar to that used for derivation of the momentum equation, it is possible to derive a separate energy equation:

\[
\frac{dD}{dx} = \frac{S_o - S_e - Q q_x (3\alpha - \beta_r)}{1 - \alpha Q^2 / g A^2 D} \tag{7}
\]

Where:

\( \alpha = \frac{1}{2} \int_A V^2 U \, dA / \frac{1}{2} \bar{U}^3 A \), an energy coefficient in the \( x \) direction.

\( \beta_r = \frac{1}{2} \int_0^{V_B} V_B^2 q \, dD / \frac{1}{2} \bar{U}^2 q_x \), an energy coefficient for inlet flow.

\( V = \) resultant velocity in the main channel

\( V_B = \) resultant velocity entering the main channel at the boundary, \( \pm B \).

\( q_x = \int_D q \, dD \)

\( S_e \) is the slope of the energy loss curve
Mixing losses: Following the development of equations in Reference 21, it is possible to derive a useful set of mixing loss coefficients which serve as a measure of the energy loss due to shock, mixing or turbulence. Such losses are accounted for separately from those of boundary friction and these appear whenever there are local differences of velocity within the flow, either in direction or magnitude.

\[ w \ h_{\text{mix}} Q_T = w \ h_{m1} Q_M + w \ h_{m2} Q_L \]  

where: \( Q_T \) = total flow  
\( Q_M \) = flow in main channel  
\( Q_L \) = lateral flow into main channel so that \( Q_T = Q_M + Q_L \) at any point downstream of introduction of \( Q_L \)  
\( h_{\text{mix}} \) = system head loss  
\( h_{m1} \) = head loss in main channel  
\( h_{m2} \) = head loss in lateral flow

\[ h_{\text{mix}} = h_{m1} \frac{Q_M}{Q_T} + h_{m2} \frac{Q_L}{Q_T} \]

\[ \frac{h_{\text{mix}}}{v_T^2} = \xi_{\text{mix}} = \frac{h_{m1}}{v_T^2} \frac{Q_M}{Q_T} + \frac{h_{m2}}{v_T^2} \frac{Q_L}{Q_T} \]

\[ \xi_{\text{mix}} = \xi_{m1} \frac{Q_M}{Q_T} + \xi_{m2} \frac{Q_L}{Q_T} \]  

where \( v_T \) is the average velocity downstream and \( \xi \) is a mixing loss coefficient
The detailed development of equations 6, 7, 8, and 9 will appear in a dissertation being written by Mr. Soong and in a publication to be taken from the dissertation.

Figures 2, 3, and 4 show how the mixing loss coefficients vary with \( \frac{Q_m}{Q_p} \) for the test configuration representing the confluence of two wide rivers. Of special interest is the negative region for \( \varepsilon_{mz} \) for values of \( \frac{Q_m}{Q_p} \) less than 0.2 shown in Figure 4. The implication here is that the energy levels in the main stream are a great deal larger than those in the side channel, so that the side flow is having its energy augmented by being drawn into the main channel.

Careful computations using equations 6 and 7 have shown that analysis of such mixing flows is extremely sensitive to variation of the \( \alpha \) and \( \beta \) terms. In fact, the question of apparent negative friction losses noted in the Appendix is completely resolved by introducing the measured values of \( \alpha \) and \( \beta \) into the flow equations. Such effects as velocity distributions across the width and depth of the main channel and streamwise components of lateral inflow must be measured or estimated to make energy analyses meaningful for any geometrical configuration.

RESULTS OR CONCLUSIONS

1. For the test configuration chosen, the flow was strongly two-dimensional, so it was possible to observe the effect of this geometry. It is clear that the importance of this effect is strongly dependent on the ratio of lateral inflow to main stream flow.

2. For channels having sufficient length for friction to be an important factor, or for cases where the lateral inflow is large enough to produce significant mixing losses, it is essential that both momentum and energy forms of the velocity distribution coefficients be applied. The assumption of unity for these coefficients will always produce negative energy losses for some flow conditions.

3. Any reasonable wall shear evaluation may be used as a basis for estimating the friction loss at the wall. The Manning formula is satisfactory if the wall roughness is known.

4. It is necessary in analysis to separate the momentum and energy equations as shown in the procedural section of this report. These methods will be applied to other geometrical configurations at a later date.

LIST OF PUBLICATIONS

None so far. Mr. Soong is still writing his dissertation. At least two publications are contemplated: one dealing with the analytical equations which were developed by Dr. J. D. Lin, and another covering results of the experimental work.
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ABSTRACT

A detailed experimental study of the frictional effects in side channel spillways with increasing discharge was conducted using a model of the convergence of two wide channels as a test case. Careful reworking of the analysis showed that a single momentum equation as used in the past was not sufficient to explain certain anomalies. Additional equations were developed for energy and mixing losses. These equations proved satisfactory in matching experimental results.

KEY WORDS

*Spatially varied flow, *Side-channel spillways, *Open channel hydraulics, Roof gutters, Street gutters, Dam spillways
Figure 1. Velocity Distribution in the Neighborhood of the Junction.

Velocity Scale: 2 ft./sec.
FIGURE 2. HEAD LOSS COEFFICIENT

$Q_m = 0.66 \text{ cfs}$
$Q_m = 0.51 \text{ cfs}$
$Q_m = 0.40 \text{ cfs}$
Figure 3. Main stream head loss coefficient vs flow ratio

$$\xi_{m1} vs \frac{Q_L}{Q_T}$$

$$\xi_{m1} = \frac{h_{m1}}{\frac{V_i^2}{2g}}$$
APPENDIX

Preliminary Report from original proposal, dated February 24, 1971

Survey of Current Work:

An excellent "state-of-the-art" summary of this problem is given in Reference 1. Excerpts from this are cited in the following paragraphs.

A. Basic Principles and Assumptions. Spatially varied flow has a non-uniform discharge resulting from the addition of water along the course of flow. The added water will cause disturbance in the energy or momentum content of the flow. As a result, the hydraulic behavior of a spatially varied flow is more complicated than that of a flow of constant discharge. Furthermore, the hydraulic behavior of spatially varied flow with increasing discharge is different in certain aspects from that of similar flow with decreasing discharge.

B. Flow with Increasing Discharge. In this type of spatially varied flow, an appreciable portion of the energy loss is due to the turbulent mixing of the added water and the water flowing in the channel. In most cases, this mixing is of relatively high magnitude and uncertainty. Because of the resulting high and uncertain losses, the momentum equation will be found more convenient than the energy equation in solving this problem. From a practical viewpoint, the high energy loss seems to make channels designed for such spatially varied flow hydraulically inefficient, but physical circumstances sometimes make the use of such structures desirable.

A substantially correct form of the fundamental differential equation for spatially varied flow with increasing discharge was probably first established by Hinds [2] for the design of lateral spillway channels. A more complete equation, however, was developed by Favre [3,4], including a friction term and a component of inflow velocity in the direction of the axis of the channel. The methods developed by Hinds and Favre are applicable to any channel, prismatic and nonprismatic, but the procedure requires a step computation with successive approximations. For prismatic rectangular channels with uniform inflow throughout the channel length, the differential equation of the flow has been integrated by Camp [5] and Li [6]. Li also treated prismatic channels of sloping walls. Theoretical and experimental studies of the flow were also made by De Marchi [7], Citrini [8], Forchheimer [9], Schoklitsch [10], and others. In practical applications, the theory has covered a variety of problems, from the study of flow in roof gutters [11] to the design of wash-water troughs in water treatment plants [12,13] and of side-channel spillways on dams.

An important type of surface flow encountered frequently in engineering deals with runoff from a plane surface such as a highway or roof as the result of rainfall. The theory of such flows was first studied by Keulegan [14], and
the equation thus derived was applied to experimental data by Izzard [15]. For flow on a road surface, a comprehensive analysis was performed by Iwagaki [16]. These studies were extended to flow behavior in channels by Keulegan [17].

C. In the derivation of the spatially-varied-flow equation, the following assumptions will be made:

1. The flow is unidirectional. Actually, there are strong cross currents present in the form of spiral flow, particularly in lateral spillway channels. The effects of these currents and of the accompanying turbulence cannot be easily evaluated, but will be included in computations if the momentum principle is used. The lateral unevenness of the water surface, as a result of cross currents, can be ignored.

2. The velocity distribution across the channel section is constant and uniform; that is, the velocity distribution coefficients are taken as unity. However, proper values of the coefficients may be introduced, if necessary.

3. The pressure in the flow is hydrostatic; that is, the flow is parallel. The flow at the outlet, however, may be curvilinear and deviate greatly from the parallel-flow assumption if a hydraulic drop occurs. In such cases, proper values of the pressure-distribution coefficients may be introduced, if necessary.

4. The slope of the channel is relatively small; so its effects on the pressure head and on the force on channel sections are negligible. If the slope is appreciable, corrections for these effects may be applied.

5. The Manning formula is used to evaluate the friction loss due to the shear developed along the channel wall.

6. The effect of air entrainment is neglected. A correction, however, may be applied to the computed result when necessary.

D. Dynamic Equation for Spatially Varied Flow. The discussion is given separately for flow with increasing discharge and flow with decreasing discharge.

1. Flow with Increasing Discharge. Referring to the lateral spillway channel in Fig. 1, the momentum passing section 1 per unit time is

\[
\frac{w_{QV}}{g}
\]

where \( w \) is the unit weight of water, \( Q \) is the discharge, and \( V \) is the velocity. Similarly, the momentum passing section 2 per unit time is

\[
\frac{w(Q + dQ)(V + dV)}{g}
\]
where \( dQ \) is the added discharge between sections 1 and 2. The momentum change of the body of water between sections 1 and 2 is, therefore, equal to

\[
\frac{w(Q + dQ)(V + dV)}{g} - \frac{wQV}{g} = \frac{w(QdV + (V + dV)dQ)}{g}
\]

Let \( W \) be the weight of the body of water between the sections. The component of \( W \) in the direction of flow is

\[
W \sin \theta = wS_o(A + 1/2dA)dx = wS_oA dx
\]

where slope \( S_o \) is equal to \( \sin \theta \) and the term containing the product of differentials is dropped.

The friction head between the two sections is equal to the friction slope \( S_f \) multiplied by the length \( dx \) or

\[
h_f = S_f dx
\]

where the friction slope may be represented by the Manning formula as

\[
S_f = \frac{V^2n^2}{2.22R^{4/3}} = \frac{Q^2n^2}{2.22A^2R^{4/3}}
\]

The frictional force along the channel wall is equivalent to the pressure due to friction head multiplied by the average area or

\[
F_f = w(A + 1/2dA)S_f dx = wAS_f dx
\]

where the product of the differentials is dropped.

The total pressure on section 1 in the direction of flow is equal to the unit hydrostatic pressure at the centroid of the water area \( A \) multiplied by the area, which is equivalent to the moment of \( A \) about the free surface.
multiplied by \( w \), or

\[
P_1 = wzA
\]

where \( z \) is the depth of the centroid of \( A \) below the surface of flow. Similarly, the total pressure on section 2 is

\[
P_2 = w(z + dy)A + \frac{w}{2}dA\ dy
\]

where \( dy \) is the difference between the depths of the two sections 1 and 2. Neglecting the term containing differentials of higher order,

\[
P_2 = w(z + dy)A
\]

The resultant hydrostatic pressure acting on the body of water between sections 1 and 2 is

\[
P_1 - P_2 = -wA\ dy
\]

Equating the momentum change of the water body to all the external forces acting on the body,

\[
\frac{W}{g} [QdV + (V + dV) dQ] = P_1 - P_2 + w\sin\Theta - F_f
\]  

Neglecting \( dV \) \( dQ \) and substituting in the above equation all expressions for external forces expressed previously,

\[
dy = -\frac{1}{g} V dV + \frac{V}{A} dQ + (S_o - S_f) \ dx
\]  

Since \( V = Q/A \) and \( V + dV = (Q + dQ)/(A + dA) \), the above equation becomes

\[
dy = -\frac{V}{g} \frac{2AdQ - QdA + dAdQ}{A^2 + AdA} + (S_o - S_f) \ dx
\]  

Neglecting \( dA \) in the denominator and \( dA \) \( dQ \) in the numerator, and simplifying,

\[
\frac{dy}{dx} = \frac{S_o - S_f - 2Qq_*/gA^2}{1 - Q^2/gA^2 D}
\]

where \( q_* = dQ/dx \), or the discharge per unit length of the channel, and \( D \) is the hydraulic depth. If nonuniform velocity distribution in the channel section is considered, an energy coefficient can be introduced in the equation, or

\[
\frac{dy}{dx} = \frac{S_o - S_f - 2aQq_*/gA^2}{1 - a Q^2/gA^2 D}
\]

E. If one accepts the six assumptions given in part C above, the analysis is correct. The questions raised by experimental studies (Keulegan, Scottron,
both unpublished, about 1960) concern the validity of assumptions 1, 2 and 5. Without detailed studies of flow velocities, pressure distributions, and flow directions at a large number of cross-sections in the length of the channel, such anomalies as negative friction losses appear in Equation (5).

It is also clear that large variation in local velocity at any channel section will invalidate Equation (2). Studies of the velocity distributions across typical channel sections will allow application of a momentum coefficient $\beta$ to a modified Equation (2).

List of References Cited:


